Playground Physics: Determining the Moment of Inertia of a Merry-Go-Round

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A playground can provide a valuable physics education laboratory. For example, Taylor et al.[1] describe bringing teachers in a workshop to a playground to examine the physics of a seesaw and slide, and briefly suggest experiments involving a merry-go-round. In this paper, we describe an experiment performed by students from a Society of Physics Students organization and their faculty advisor on a merry-go-round at a local park. The goal of the activity was for everyone to gain a greater understanding of the concepts of angular velocity, centripetal acceleration, moment of inertia, and conservation of angular momentum through their own personal experience—and to have fun, too.

The official objective of this field trip was to find the moment of inertia of the merry-go-round (MGR). The procedure we developed was for two of us to start off in the center of the MGR and then quickly move to the edge of the MGR, or vice versa (see Fig. 1). The physics describing this situation is just the conservation of angular momentum, or

$$L_i = I_i\ \omega_i = I_f\ \omega_f = L_f, \text{ or} \qquad (1)$$

$$(I_{MGR} + m_1 r_{1i}^2 + m_2 r_{2i}^2)\ \omega_i = (I_{MGR} + m_1 r_{1f}^2 + m_2 r_{2f}^2)\ \omega_f \qquad (2)$$

$$I_{MGR} = [\ (m_1 r_{1f}^2 + m_2 r_{2f}^2)\ \omega_f - (m_1 r_{1i}^2 + m_2 r_{2i}^2)\ \omega_i\ ]\ /\ (\omega_i - \omega_f). \qquad (3)$$

We developed two ways of determining the angular velocity ω of the MGR: (a) measure the centripetal

Fig. 1. The moving inward experiment: (a) Two of us stand on the outer edge of the MGR while a third person sits on the inside to start the data collection and tell the others when to start coming in. A fourth person spins up the MGR (and stops it after the experiment). (b) Once we have moved inside as much as we are able to given space constraints and our ability to pull ourselves inward.

acceleration ($a_c = r\omega^2$) at a known radius using a low-g accelerometer connected to a Vernier LabPro and TI-83+ calculator (see Fig. 2); and (b) analyze video of the experiment with iMovie[2] and determine the period ($T = 2\pi/\omega$) of its rotation. The accelerometer was duct-taped at the edge of the MGR (r = 1.49 m), while the LabPro was positioned in the center of the MGR. During the moving inward experiments, an additional person sat in the middle of the MGR to start the LabPro—this person's mass added a negligible contribution (of order 1 kg · m^2) to the moment of inertia of the MGR.

We did four runs before we began to feel queasy from the experience—one where we moved outward, and three where we moved inward. We analyzed two of these four runs since the first two of the moving inward experiments were complicated by our difficulties in moving inward when the MGR was rotating. Table

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Table I. A comparison of ω and I_{MGR} determined from the centripetal acceleration and from video analysis.

	Calculated from a_c	Observed in video	% Difference
Moving out:	$\omega_i = 2.63 \pm 0.08$ rad/s $\omega_f = 1.63 \pm 0.05$ rad/s $I_{MGR} = 290 \pm 40$ kg · m^2	$\omega_i = 2.65 \pm 0.08$ rad/s $\omega_f = 1.63 \pm 0.05$ rad/s $I_{MGR} = 285 \pm 40$ kg · m^2	1% 0% 2%
Moving in:	$\omega_i = 2.46 \pm 0.07$ rad/s $\omega_f = 3.38 \pm 0.10$ rad/s $I_{MGR} = 410 \pm 70$ kg · m^2	$\omega_i = 2.58 \pm 0.08$ rad/s $\omega_f = 3.70 \pm 0.11$ rad/s $I_{MGR} = 350 \pm 60$ kg · m^2	5% 9% 16%

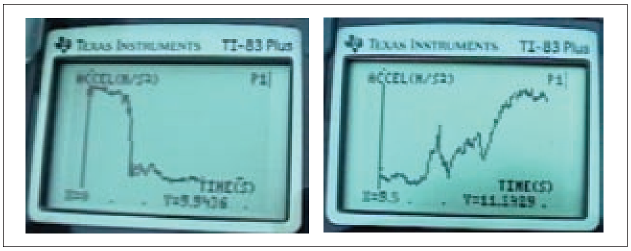

Fig. 2. The centripetal acceleration-vs-time plots for our (a) moving outward and (b) moving inward runs for a 15-s interval. Notice that the transition in (a) is cleaner than in (b) since it was very easy to move outward on the MGR but moving inward was quite a struggle.

I summarizes the results from these two experimental runs. The uncertainty in ω calculated from a_c originates from the variation in a_c, while the uncertainty in ω calculated from the video originates from the uncertainty in measuring the angular displacement of the MGR between video frames. The uncertainty in I_{MGR} is due to the uncertainty in ω and the uncertainty in determining the radius of the center of mass of each person in the video.[3]

The ω's determined using our two methods were close (an average difference of 4%), and some of that may be due to not determining ω at the same instant on the LabPro and in the video. All but one of the I_{MGR} values agree within the experimental uncertainty, but calculating a larger I_{MGR} for moving inward than moving outward could originate from ignoring friction acting on the MGR. Friction would add a torque τ that would affect our calculations for moment of inertia as:

$$I_{MGR} \text{ (actual)} = I_{MGR} \text{ (calculated)} + \tau\Delta t/(\omega_i\text{-}\omega_f), \quad (4)$$

where Δt is the time interval between measurements of ω_i and ω_f. If we assume that all of the difference between the values of I_{MGR} calculated from the video were due to friction, we would require a frictional torque of 5 Nm. With a friction torque of that magnitude, the MGR would take approximately two minutes to come to a full stop from an angular velocity of 2 rad/s (a typical value in our experiments), which seems reasonable. Unfortunately, we did not videotape a long enough time to measure the angular deceleration due to friction, and we could not go back and test this prediction since the merry-go-round was removed from the park that summer.

Conclusion

Both the accelerometer and video camera seem equally useful in calculating the angular velocity of the merry-go-round but since the positions of the people on the MGR (especially the outer positions) are critical, the video camera is a better single tool for doing the experiment. This field trip to experiment on the merry-go-round was a great way to experience rotational motion and provide an opportunity to test physics principles outside the laboratory. You can see a QuickTime movie of our experiment at http://physics.bgsu.edu/~vanhook/mgrexperiment. We encourage you to show this to your students if you can't bring them to a playground to do the experiment for themselves. Some key experiences we had were:

- We were surprised how difficult it was to move inward when the MGR was rotating. Quite a large force was required on our part to stay on the MGR or move inward ($F = ma_c = m\omega^2 r$), which in our experiment was approximately equal to our weight.
- It was very real to us that we were doing work—exerting a force over a distance—when we tried to move inward as the MGR was rotating.
- We discovered that we wanted to be the first person to move inward, since it was even harder for the second person because the MGR's angular velocity had increased due to conservation of angular momentum.
- We discovered that a MGR has quite a lot of ro-

tational inertia and in practice that means that it's not easy to stop. (See the video for what happened to one of us when he first tried to stop the MGR rotating.)

Unfortunately, due to insurance issues more and more parks are retiring their merry-go-rounds. For example, the merry-go-round that we used was removed from the park a few months after we filmed our experiment.

References

1. Richard Taylor, David Hutson, Wesley Krawiec, Jhone Ebert, and Robin Rubinstein, "Computer physics on the playground," *Phys. Teach.* **33,** 332–337 (Sept. 1995).
2. Part of the iLife suite on the Macintosh by Apple computer (http://www.apple.com).
3. The uncertainty is calculated according to standard procedures for propagating random errors in a calculation, as described in depth by J.R. Taylor, *An Introduction to Error Analysis* (University Science Books, Herndon, VA,1982), pp. 56–57.

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